**Bioeconomics of fishing with seasonal variations in catchability**

Assignment for SOK-8004, Spring 2013

Øystein Hermansen

**Introduction**

Fisheries related bioeconomic theory and models have been widely employed since their development in the 1950ies and have provided insight on a number of aspects regarding the utilization and dynamics of renewable resources. Early models focused on explaining the economic and behavioral drivers that led to overutilization and depleted fish stocks (Gordon 1954, Scott 1955). Refinements of the models showed the considerable rents that could be acquired if fishing effort is reduced and the stock allowed growing to optimal levels.

A sound aim for management of fish resources is to maximize the rents generated from their utilization. Insights from biology and economics in particular have led to considerable improvements in the management of many fish stocks through TACs and various forms of ownership incentives such as ITQs. Management, however, often has a time perspective limited to one year to another and is thus not explicitly taking into account the relatively strong seasonal variations in parameters such as growth, quality characteristics and costs of harvesting. Optimal resource utilization is likely to depend on these variations. Hence, the timing of harvest is important for rent maximization, named the "time of capture problem" by Wilson (1982).

Common for most modeling exercises is the reliance on simplification and assumptions. Prime assumptions in bioeconomic models are some representation of the stock dynamics and the harvesting dynamics. A highly popular model for the former is the logistic growth function (Verhulst 1838). The latter is often represented by the Schaefer production function (Schaefer 1954), where harvest is dependent on the effort applied, the stock level and a parameter describing how easy the fish are to catch. Next, one often has to make assumptions about the parameters going into these models. In most cases these are rather simplistic, in order to ease the interpretation of the model results and to provide insights on the basics of the systems.

Examples of such simplifications are constant prices of fish and effort, constant catchability and constant intrinsic growth rates. In real life, prices may depend on the level of supply and the harvesting costs may depend on stock levels. Economists are generally aware of the implications of their simplifications, and the seasonal variations have received some attention in the literature, and several bioeconomic models have been developed taking seasonal characteristics into account. Flaaten (1983) included cyclical growth rates in a Verhulst-Schaefer growth function. He found optimal harvest also to be seasonal and that the optimal season was shorter than the open access alternative, ending at the same time, when stock size crossed into unprofitable level, but starting later. Considerably more attention has been paid to investigating the effect of variations in sales prices. Bjørndal (1988) and Anderson (1989) analyzed the problem assuming prices are a function of individual size. Other studies such as Anderson & Martinez-Garmendia (2001) and McConnell & Strand (2000) have investigated other product characteristics. These studies show that harvests should be delayed in order to sell products that obtain higher prices. Larkin & Sylvia (1999) found that rents could be increased from altering the season opening date in the Pacific whiting fishery by taking conversion yield into account. Larkin & Sylvia (2004) further developed their model taking other seasonally variable quality characteristics through a hedonic price model and again demonstrated the potential for generating more rent through more optimal utilization.

The key lessons from these studies are that bioeconomic models, if not taking these characteristics into account, can underestimate the potential rent generation, and, perhaps more importantly, give suboptimal advice for how the resource should be utilized.

In most bioeconomic models of fisheries, catchability is also assumed to be constant over time. Many fisheries, however, do not express such a simplistic behavior, and have a catchability that varies seasonally and spatially (Arreguin-Sanchez 1996). This may be a feature that stems from a number of sources. Many stocks migrate between areas and over large distances. This will certainly result in seasonal variations at given locations. The behavior of the fish may also vary over time, sometimes the fish are spread out while at other times the fish are aggregated. Feeding behavior may change over the year – at certain periods the fish may be more interested in feeding, creating a larger probability of taking bait. The last example illustrate well that the seasonal changes in catchability may often be gear specific.

The aim of this paper is to analyze how seasonal variations in catchability influences optimal stock level and harvesting pattern over time. The analysis is carried out using a Gordon-Schaefer type bioeconomic model of a fishery in the computer program Mathematica version 9. We investigate the results for different management regimes; open access and ITQ. In addition we discuss the optimum adjustment path in moving towards equilibrium.

The paper is structured as follows. The next section describes how a basic bioeconomic model changes due to the introduction of seasonality in catchability. The third section describes the actual model and assumptions employed in the analysis. In the final section the results are presented and discussed.

## General model

The following section presents the general model and method that is applied to analyze the research question for this paper. To determine optimal stock levels and harvesting patterns, one needs a benchmark to compare alternatives against. In fisheries, a number of political aims are proposed that are often conflicting; e.g. profits, labour, food supply, welfare. Although the latter is likely to best capture society's wishes from fisheries, this would be difficult to quantify. As a reasonable proxy, and very common in fisheries economics, we select economic rent generation as the objective for this study.

We are thus faced with a problem of how to utilize a given fish resource to maximize the profits stemming from this utilization. This will be our objective function. Here we assume that the resource manager controls the utilization. His control is however limited to the fishing side – he cannot influence how the fish stock grows. This is determined by biological and ecological variables. Even in fishing he may have less than perfect control as fishing cannot be less than 0 and likely have a maximum amount of fishing effort available. This implies that the system has a number of constraints – placing a cap on the profits that are possible to extract from utilizing the resource.

Systems consisting on an objective function and a number of constraints can be optimized through a number of methods, depending on the specification of the system. Flaaten (2011) shows how standard investment theory can be applied to a profit maximization problem in fisheries. The Euler function can also be utilized for simple problem formulations (Clark 2010). For this study we will employ the theory on optimal control, developed in Pontyagrin *et al.* (1962) and further developed for fisheries in the famous seminal paper by Clark and Munro (1975). Here, they arrived at investment rules that describe optimal stock and harvest levels for different model formulations.

The fisheries model can be generalized through equations 1 and 2. Here, profits, π, is a function of the stock level, *x*, and harvest, *h*. Present values (PV) are obtained through discounting profits by the discount rate, δ. Here, *x* is the state variable and *h* is the control variable.

$$\max PV = \int_0^\infty \pi\big(x(t), h(t)\big) e^{-\delta t} dt \qquad \text{(Eq. 1)}$$

$$s.t. \quad x'(t) = f(x(t), h(t)) \qquad \text{(Eq. 2)}$$

The problem is to find the controls that maximize the discounted profit stream from the resource, while the state variable changes as described in the differential Eq. 2. This is done through finding the Hamiltonian to the system of equations and determining the switching function. By utilizing Pontryagins maximum principles on the switching function, one can derive the optimum for the state and control variable. In fisheries these are defined by the “modified golden rule” developed in Clark and Munro (*op cit*). In autonomous and linear problems, the rule is defined as in Eq. 3.

$$G'(x) - \frac{c'(x)G(x)}{p - c(x)} = \delta \qquad \text{(Eq. 3)}$$

Eq. 3 has a clear economic interpretation. On the right hand side we have the social rate of discount. At optimal stock level, this is equal to the interest rate obtained from the fish stock. The latter is composed of two elements. *G′(x)* represents the marginal product of the resource and the second element is known as the marginal stock effect, representing the return from the change in harvesting costs associated with the stock level. Eq. 3 can be solved for *x* to give optimum stock level as shown in Eq. 4.

$$x^* = K/4\left[\left(\frac{c}{pqK} + 1 - \frac{\delta}{r}\right) + \sqrt{\left(\frac{c}{pqK} + 1 - \frac{\delta}{r}\right)^2 + \frac{8c\delta}{pqKr}}\right] \qquad \text{(Eq. 4)}$$

## A basic model with seasonal catchability variation

This section specifies the model employed in this study. First and foremost, a description of how the fish stock would develop without fishing is necessary. The complexity of nature is vast, and we have to rely on strong simplifications of the actual processes taking place (Loehle 2006). For this study we employ the logistic growth model originally developed by Verhulst (1838) and shown in Eq. 5 to describe how the biomass of the stock changes over time. This formulation supposes that the growth of the stock is dependent on the intrinsic growth rate, r, the level of the stock, x and the carrying capacity of the environment, K.

$$G(x) = {dx}/{dt} = rx(1-\frac{x}{K}) \qquad \text{(Eq. 5)}$$

Thus, growth follows a quadratic relationship, and is thus density dependent. Here, r and K are assumed constant, although the former most often has seasonal variations, as discussed from an optimal economics perspective in Flaaten (1983). The solution of this differential equation provides the stock size as a function of time as shown in Eq. 6.

$$x(t) = {K}/{1+ce^{-rt}} \quad , where\ c = {K - x0}/{x0} \qquad \text{(Eq. 6)}$$

In fish stocks that are economically profitable to harvest, stock size is also influenced by this activity. From Eq. 3 we have to subtract *h*(*t*) to attain the net change in the stock. The Schaefer production function is popular to describe the effect of fishing and is shown in Eq. 7 below. Here, q is catchability and E fishing effort. With this representation, we assume there is a linear relationship between each of the inputs and harvest. Usually, catchability is assumed constant in bioeconomic models. In reality, and for most fisheries, this varies considerably over the course of the year. Here, we will treat it as a deterministic cyclic variable. This will be further specified in the following results section of the paper.

$$h(t) = q(t)E(t)x(t) \qquad \text{(Eq. 7)}$$

Fishing costs and revenues are naturally key components of the model. Although prices often vary with a number of features, particularly time and quality characteristics as discussed in e.g Larkin & Sylvia (2004), for simplicity we here assume it constant. Unit cost of effort, *c*, is also assumed constant here. Transforming this to the unit cost per harvest unit, utilizing the harvest function yields a unit cost that varies with catchability and stock size as shown in Eq. 8. Revenues and costs are thus given by Eq. 9 and 10, respectively.

$$c(x) = c/(q(t)\,x(t)) \qquad \text{(Eq.8)}$$

$$TR(t) = p\,h(t) \qquad \text{(Eq. 9)}$$

$$TC(t) = c(q(t), x(t))\,h(t) \qquad \text{(Eq. 10)}$$

We can now define the optimization problem through its objective function and constraints as shown in equations 11 - 14. For the purposes of this study, we do not impose any restrictions on effort levels.

$$\max PV = \int_0^\infty (p - c[q(t), x(t)]h(t)e^{-\delta t}dt \qquad \text{(Eq. 11)}$$

S.t. $$dx/_{dt} = G(x) - h(t) \qquad \text{(Eq. 12)}$$

$$x(t) \geq 0 \qquad \text{(Eq. 13)}$$

$$h(t) = q(t)E(t)x(t) \qquad \text{(Eq. 14)}$$

The Hamiltonian, *H*, and switching function, *σ(x,t)*, stemming from this problem is given in Eq. 15 and 16. *H* can be interpreted as the change in present value stemming from the cash flow from fishing and the change in value of the stock. The adjoint variable $\lambda(t)$ is the shadow price of the stock.

$$H = e^{-\delta t}[(p - c(q, x)) - \lambda]h + \lambda G(x) \qquad \text{(Eq. 15)}$$

$$\sigma(x, t) = e^{-\delta t}[(p - c(q, x)] - \lambda \qquad \text{(Eq. 16)}$$

We are interested in the area where the switching function is equal to 0. For the other options the bang-bang approach or investing/disinvesting in the stock at the maximum rate are the relevant solutions.

$$\lambda(t) = e^{-\delta t}(p - c[q(t), x(t)]) \qquad \text{(Eq. 17)}$$

According to Pontryagins maximum principle, global maximum is found where $\frac{d\lambda}{dt} = -\frac{\partial H}{\partial x}$. We proceed by performing the necessary partial derivatives.

$$\frac{\mathrm{d}}{\mathrm{dt}}\lambda(t) = e^{-\delta t}[-\delta\big(p - c(q, x)\big) - c'(q, x)\frac{dq}{dt} - c'(q, x)\big(G(x) - h(t)\big)] \qquad \text{(Eq. 18)}$$

$$-\frac{\partial H}{\partial x} = e^{-\delta t}[c'(q, x)\,h(t) - \big(p - c(q, x)\big)G'(x) \qquad \text{(Eq. 19)}$$

Next, the derivatives are equated and simplified, yielding the modified golden rule as shown in Eq. 20. We can see that it differs slightly from the golden rule shown in Eq. 3 for a Gordon-Schaefer model stock with constant *q* and also where *q* is a function of *x* only (Clark 2011: p.67). In this situation, we get a third term in addition to the marginal productivity of the stock and the marginal stock effect. The third term can be called the marginal catchability effect and captures the differences in return when fishing at different catchability.

$$\delta = G'(x) - \frac{c'(q,x)G(x)}{p-c(q,x)} - \frac{c'(q,x)\frac{dq}{dt}}{p-c(q,x)} \qquad \text{(Eq. 20)}$$

## Results

We started out by defining the seasonal variation in catchability. For simplicity, we assumed this to be periodical oscillations around a mean with a period of one year. We chose the formulation $q(t) = \frac{1}{50}(2 + \mathrm{Sin}[2\pi t])$, shown graphically in Figure 1 to represent the seasonal catchability. The resulting *q*-function bears close resemblance to the pattern deduced for the trawl fishery for Atlantic cod (*Gadus morhua*) as described in Eide *et al* (2003).

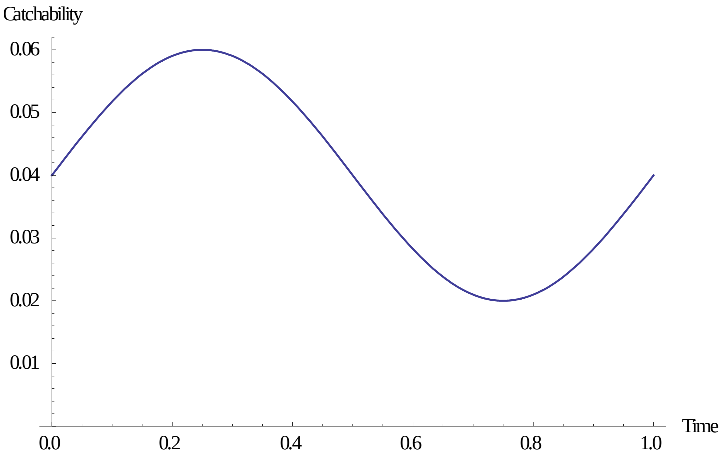



**Figure 1 Catchability assumption for the analysis**

As noted, with costs being linear in effort, costs of fishing per unit harvest is dependent on both catchability and stock size. We have assumed a stock that is described by $r$ = 0,3, $K$ = 100 and an initial stock size, *x0* of 26,5. Price of fish per unit harvest, $p$, is 10 and cost per unit effort, $c$, is 4. For illustrative purposes, we have shown how the seasonal variations in catchability influences costs per unit harvest in Figure 2. Stock size is assumed to be the pristine stock size. Catchability has a clear impact on the profits from fishing at different times of the year; at peak “season”, costs are less than half of the low “season”. This is in stark contrast to the constant catchability case, where unit costs decrease slightly due to the effect of a growing stock.

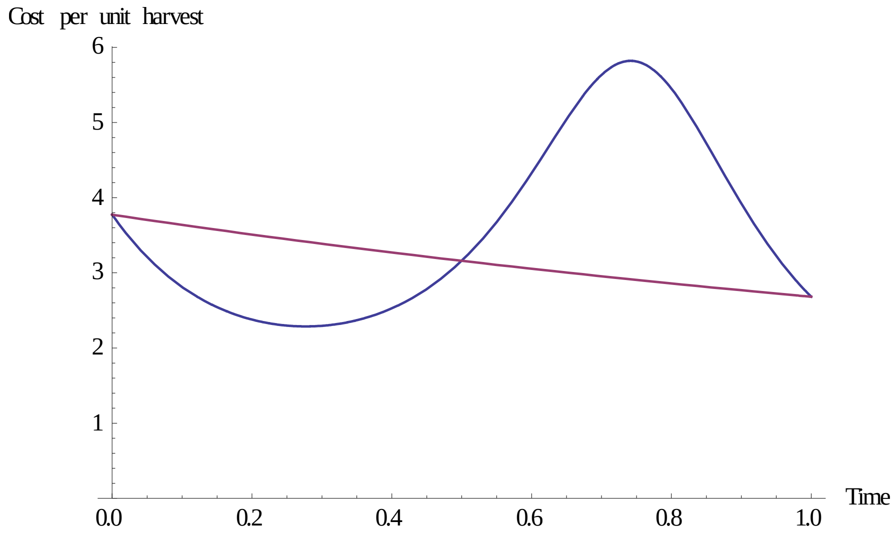



**Figure 2 Costs per unit harvest as a function of time**

Figure 3 shows the results for optimal stock and harvest during one period. Due to problems in obtaining the solution of Eq. 20 with respect to optimal stock level, we have employed the solution for the golden rule without the *dq/dt* variable. For reference and comparison we have also plotted the stock development with no fishing. In this case, we have assumed the stock to start at the optimal level. We have seen that the problem was linear in the control variable. Hence, according to Clark (2010), the optimal path from a suboptimal starting stock to the optimum will either be obtained by employing maximum effort or no fishing, depending on the starting point being above or below optimum. This is known as the bang-bang approach.

We have plotted only one period, as the other periods are alike for optimal stock and harvest. The pristine stock would naturally grow towards *K*, as shown in the upper line. Whereas the pristine and optimal stocks are represented in biomass, harvest is a flow. Hence, we have chosen to illustrate it by the annualized catches that would be attained if catches are sustained at the same level. The figure shows that the optimal harvest starts relatively high and decreases with time. At $t = t_{stop}$, harvest is stopped before it is initiated again at $t = t_{start}$. From here it is increased rapidly until reaching its maximum at $t = t_{max}$. The period where catches are less than zero is likely to be an artifact of not being able to solve the golden rule for optimal catch properly and employing the reduced golden rule instead.

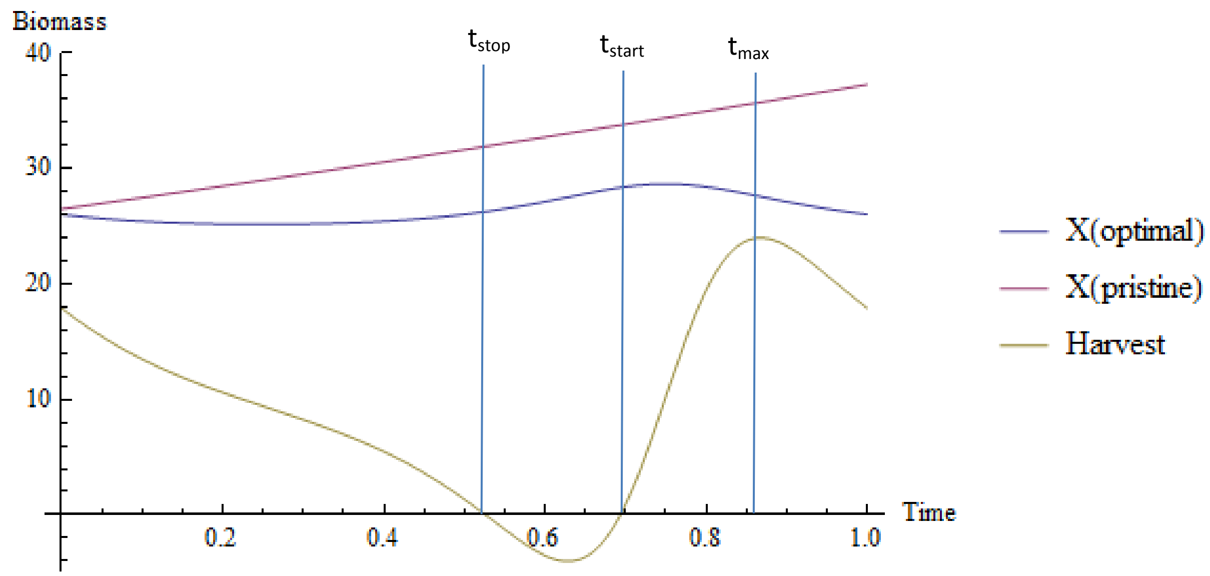



Figure 3 Singular solution for optimal stock, pristine stock and harvest

To get a better understanding of the timing of harvest, we have plotted the growth for optimum stock, *G(x)*, against the actual change in stock, *G(x)-h,* in Figure 4. We observe that intrinsic growth is relatively stable, due to the relatively small changes in optimal biomass and the constant intrinsic growth rate. Combined with fish harvesting, a clear seasonal pattern in net stock change emerges. From $t_0$ to $t_1$, we are harvesting more than the growth of the stock. From $t_1$ to $t_4$ we are harvesting less, allowing the stock to increase in size. From $t_4$ to $t_0$ again we harvest more than stock growth.

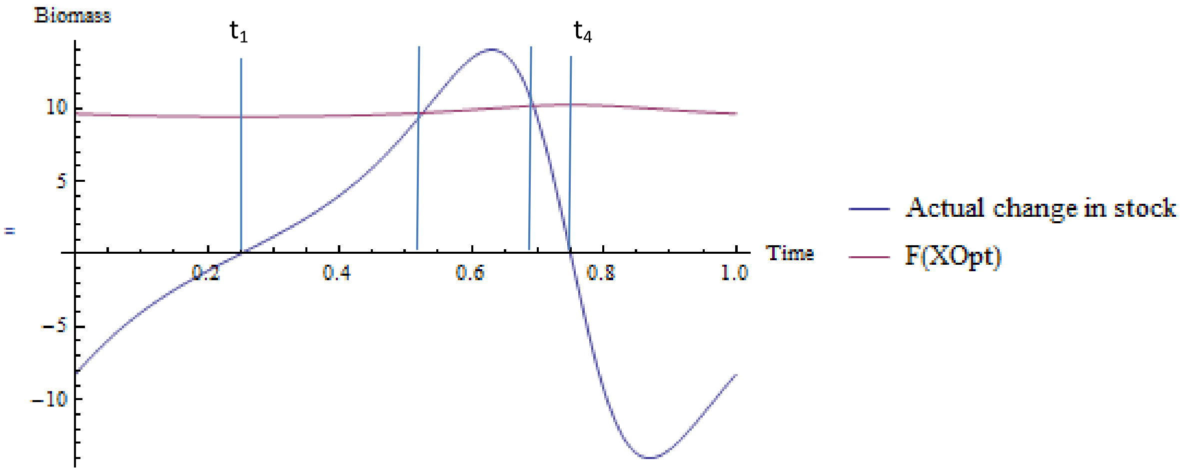



Figure 4 Actual change in stock and intrinsic growth at optimal stock

It is interesting to investigate how these results relate to the case when there is no seasonality in q, when q is a constant. We employ the average of *q(t)* and perform the calculations to obtain optimal stock level and harvest. As we know from the literature, these are constant values when no parameters are dependent on time. The results and comparison with the seasonal q results are shown in Figure 5.

Optimal stock is 26.5 and annual harvest is 9.6 with constant q. We notice that harvest for the seasonal alternative is higher to begin with, then has a period where harvests are less before they again are higher. This naturally results in a stock that is lower for the first half of the year, then higher for the second half of the year.

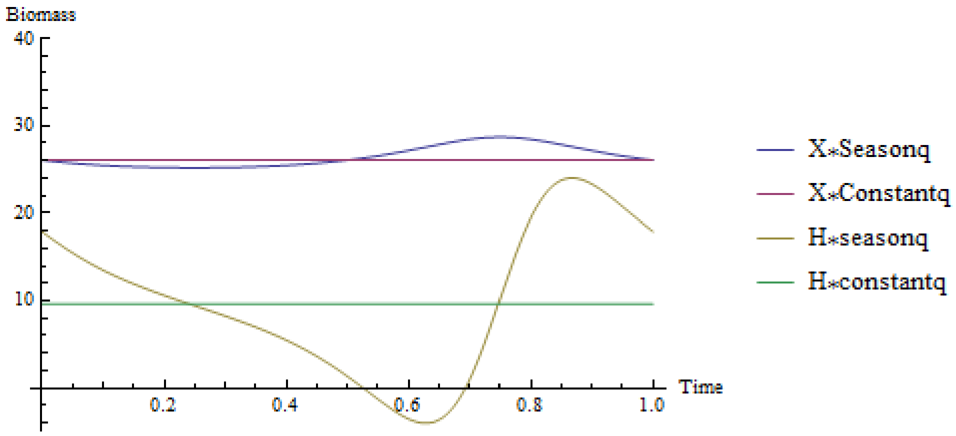
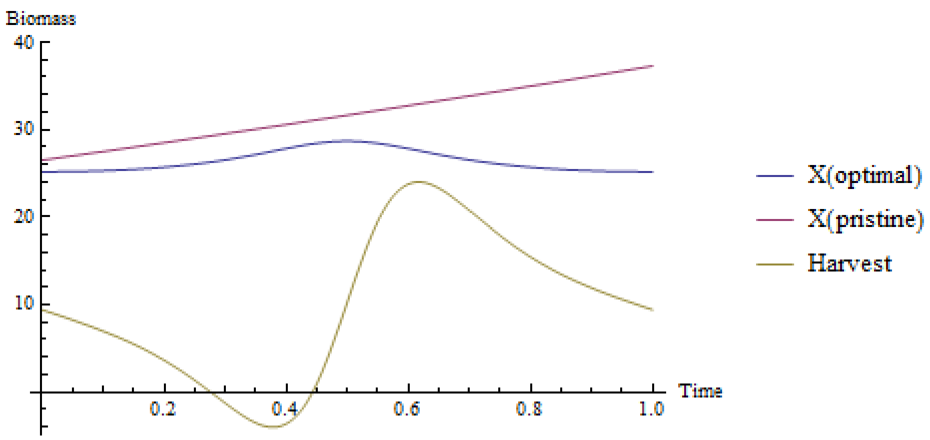



**Figure 5 Comparison of optimal stock levels and harvest rates with seasonal and constant q**

The effect of different seasonal patterns of q(t) is also interesting to investigate. Here, we replace the sine with a cosine in our *q(t)* function. This has the effect of shifting the function by 0.25 to the left on the time axis. Perhaps not surprisingly, this also shifts the optimal stock and harvest pattern correspondingly. The model predicts that fishing should be low or 0 when catchability is low and higher when it is high.




The previous two analyses have assumed a sole owner of the stock, with an aim of maximizing the value generated from fishing. Fisheries management has for most stocks developed from an open access situation to various other management systems, including some that allow for rent generation. Many fisheries, however, still bear close resemblance to the open access situation. It is therefore relevant to also analyze how our model fishery would behave with open access management. The open access equilibrium stock level is generally given by Eq. 18 (Flaaten 2011). With seasonal variations in q, it is likely that the intrinsic stock growth is not able to compensate for the increase in harvesting costs when q is decreasing. Thus, we can get a situation with no fishing until the stock has grown such that the bionomic equilibrium is reached. This is the stock level where revenues equal costs in the short run.

$x_{\infty}(t) = \frac{c}{p\, q(t)}$ (Eq. 18)

The results are shown in Figure 6. First we see relatively large variation in the bionomic equilibrium due to the seasonal variation in *q*. We assume, somewhat arbitrarily, that the stock starts at bionomic equilibrium at $t=t_0$. The stock is then fished down, as catchability increases. From $t=t_1$ fishing is no longer profitable as the growth of the stock cannot compensate for the rise in costs with less catchability. Fishing again commences at $t=t_2$, when both higher stock size and improving catchability brings a situation with profitable fishing. Fishing again ceases at $t=t_3$ and the cycle repeats in infinity.

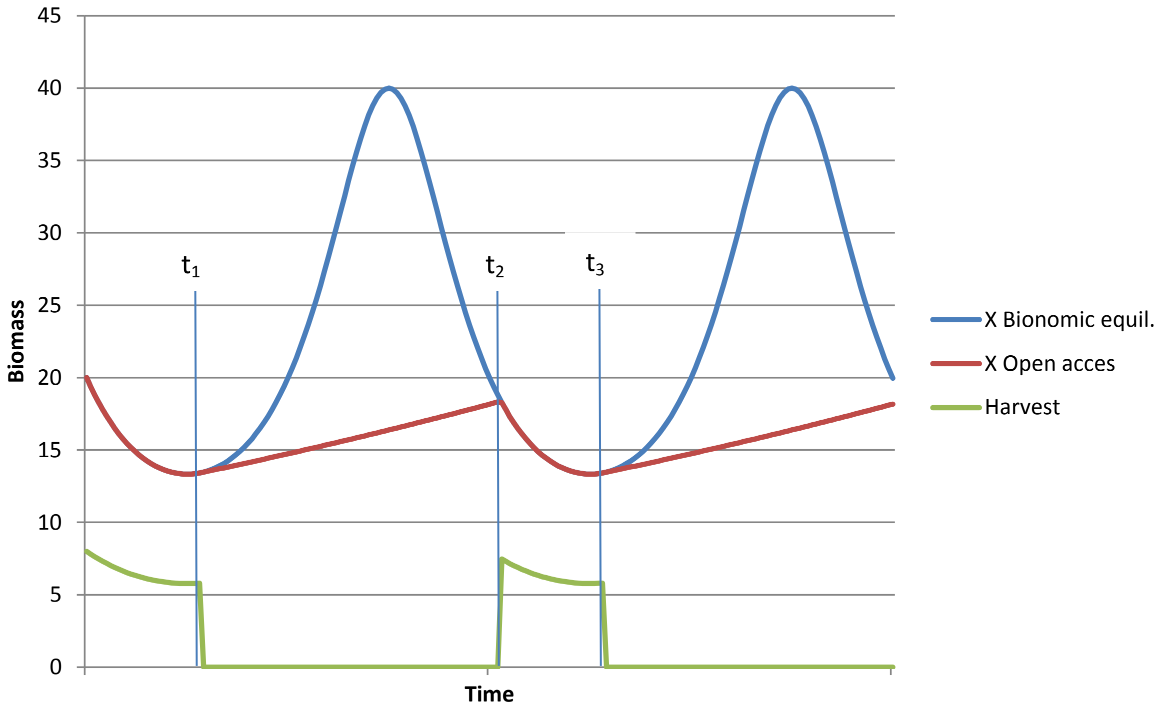



**Figure 6 Open access stock level, bionomic equilibrium, $x_{00,}$ and annualized harvest**

**Summary and discussion**

The aim of the paper was to investigate how seasonal fluctuations in the catchability parameter influences optimal stock size and utilization of a fish stock. Optimal was in this respect treated as maximum rent generation. This was analyzed employing a basic Gordon-Schaefer type bioeconomic model where the constant $q$ was replaced with a symmetric sine function with 1-year period. The resulting optimization problem was solved using optimal control theory and a modified "golden rule" was derived. This was a slight modification from the golden rule with constant $q$.

The reader should take note that the results presented are on the basis of the reduced golden rule shown in Eq. 4 due to not being able to obtain a sound solution from the golden rule derived in this paper. The results showed that optimal stock also follows a seasonal pattern, with more than the stock's intrinsic growth being harvested in given periods and vice versa. In a relatively short period of time harvesting is stopped altogether. Harvesting occurs in general when $q$ is high. In the open-access case, stock level is also found to vary across the year, but at a considerably lower level than the optimum case. Harvesting periods are shorter, and less is harvested. This is due to the depressed stock's influence on harvesting costs. No rents are of course generated.

When using insights from optimal control theory to manage complex systems such as fisheries generally are, there is a tradeoff between the realism in the system description and the need for exact solutions (Aanestad 2009). Here, we have clearly prioritized the latter, as the system is represented by highly simplified relationships. Employing more realistic representations of stock dynamics, such as seasonal growth, cohorts and recruitment, and harvesting will have influences on our results. The general result that seasonal catchability yields a more seasonal harvest level and introduces seasonal optimum stock level is likely to remain.

Another important stock-dynamic aspect disregarded in this analysis is the relationship with other stocks. A stock seldom lives in isolation from others, but depends on them for food and/or predation. Here these are assumed exogenous, and implicit in the growth assumption, but a more formal specification of predator-prey relationships, such as described in Flaaten (1988) could change the results considerably.

Uncertainty is also an aspect that is not treated in this model, as all relationships are assumed deterministic. The results are influenced by parameter values. Higher r gives increased harvest and slightly higher stock. Higher p reduces stock somewhat and gives less seasonality in catches. Higher c has the opposite effect, with higher stock and more seasonality in catches. Higher discount rate results in lower stock size and correspondingly slightly lower harvest.

**References**

Aanestad, S 2009. Essays on exploitation of natural resources. Optimal control theory applied on multispecies fisheries and fossil fuel extraction. PhD dissertation, University of Tromsø.

Anderson, JL & J Martinez-Garmendia 2001. Pricing US North Atlantic Bluefin tuna and implications for management. *Agribusiness* 17(2): 243-254.

Arreguin-Sanchez, F 1996. Catchability: A key parameter for fish stock assessment. *Reviews in Fish Biology and Fisheries* 6:221-242.

Bjørndal, T 1988. Optimal harvesting of farmed fish. *Marine Resource Economics* 6(2):139-159.

Clark, CW 2010. Mathematical bioeconomics – the mathematics of conservation. 3rd ed. Wiley.

Eide, A, F Skjold, F Olsen & O. Flaaten 2003. Harvest functions: The Norwegian bottom trawl cod fisheries. *Marine Resource Economics* 18:81-93.

Flaaten, O 1983. The optimal harvesting of a natural resource with seasonal growth. *Canadian Journal of Economics* 16(3):447-462.

Flaaten, O 1988. The economics of multispecies harvesting – theory and application to the Barents Sea fisheries. In Studies in Contemporary Economics, Springer, Berlin.

Flaaten, O 2011. Fisheries economics and management. E-book, University of Tromsø, ISBN 978-8266-026-6.

Gordon, HS 1954. The economic theory of a common-property resource. *Journal of Political Economy* 62(2):124-142.

Larkin, SL & G Sylvia 1999. Intrinsic fish characteristics and intra-season production efficiency: A management-level bioeconomic analysis of a commercial fishery. *American Journal of Agricultural Economics* 81(1):29-43.

Larkin, SL & G Sylvia 2004. Generalizing enhanced fishery rents by internalizing product quality characteristics. *Environmental and Resource Economics* 28:101-122.

Loehle, C 2006. Control theory and the management of ecosystems. Journal of Applied Ecology 43(5):957-966.

McConnell, KE & IE Strand 2000. Hedonic prices for fish: Tuna prices in Hawaii. *American Journal of Agricultural Economics* 82(1):133-144.

Pontryagin, LS, VS Boltyanskii, RV Gamkrelidze & EF Mishchenko 1962. The mathematical theory of optimal processes. Wiley-Interscience, New York.

Schaefer, MB 1957. Some considerations of population dynamics and economics in relation to the management of the commercial marine fisheries. *Journal of the Fisheries Research Board of Canada* 14(5):669-681.

Scott, A 1955. The fishery: The objectives of sole ownership. *Journal of Political Economy* 63(April): 116-124.

Verhulst, P-F 1838. Notice sur la loi que la population poursuit dans son accroissement. *Correspondance Mathematique et Physique* 10:113-121.

Wilson, JA 1982. The economical management of multispecies fisheries. *Land Economics* 58(4):417-434.